

Robust Active Perception via Data-association aware Belief Space Planning

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Abstract. We develop a belief space planning (BSP) approach that advances the state of the art by incorporating reasoning about data association (DA) within planning, while considering additional sources of uncertainty. Existing BSP approaches typically assume data association is given and perfect, an assumption that can be harder to justify while operating, in presence of localization uncertainty, in ambiguous and perceptually aliased environments. In contrast, our data association aware belief space planning (DA-BSP) approach explicitly reasons about DA within belief evolution, and as such can better accommodate these challenging real world scenarios. In particular, we show that due to perceptual aliasing, the posterior belief becomes a mixture of probability distribution functions, and design cost functions that measure the expected level of ambiguity and posterior uncertainty. Using these and standard costs (e.g. control penalty, distance to goal) within the objective function, yields a general framework that reliably represents action impact, and in particular, capable of active disambiguation. Our approach is thus applicable to robust active perception and autonomous navigation in perceptually aliased environments. We demonstrate key aspects in basic and realistic simulations.

1 Introduction

In the context of partially observable Markovian systems, planning over belief space (BSP) under some simplifying assumptions, provides scalable applications including autonomous navigation, object grasping and manipulation, active SLAM, and robotic surgery. In presence of uncertainty, such as in robot motion and sensing, the true state of variables of interest (e.g. robot poses), is unknown and can only be represented by a probability distribution over possible states, given available data. This distribution, the belief space, is inferred using probabilistic approaches based on incoming sensor observations and prior knowledge. The corresponding problem is an instantiation of a partially observable Markov decision problem (POMDP) [16]. Apart from simplifying structural assumptions – such as Gaussian noise around a given observation and motion model – state-of-the-art BSP approaches typically assume data association to be

given and perfect (see Figure 1b), i.e. the robot is assumed to correctly perceive the environment to be observed by its sensors, given a candidate action. For brevity, we shall call it DAS. In reality, the world is often full of ambiguity, that together with other sources of uncertainty, make perception a challenging task. As an example, matching images from two different but similar in appearance places, or attempting to recognise an object that is similar in appearance, from the current viewpoint, to another object. Both cases are examples of ambiguous situations, where naïve and straightforward approaches using DAS are likely to yield incorrect results, i.e. mistakenly considering the two places as same, and incorrectly associating the observed object.

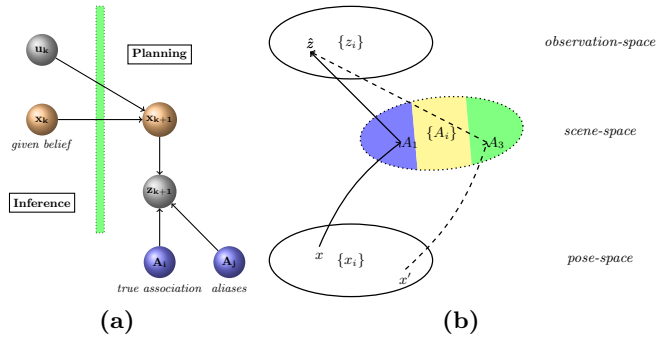


Fig. 1: (a) Generative graphical model. Standard BSP approaches assume data association (DA) is given and perfect (DAS). We incorporate data association aspects within BSP and thus can reason about ambiguity (e.g. perceptual aliasing) at a decision-making level. (b) Schematic representation of pose, scene and observation spaces. Scenes A_1 and A_3 when viewed from perspective x and x' respectively, produce the same nominal observation \hat{z} , giving rise to *perceptual aliasing*.

Thus, in presence of ambiguity, DAS may lead to incorrect posterior beliefs and as a result, to sub-optimal actions. More advanced approaches are therefore required to enable reliable operation in ambiguous conditions, approaches often referred to as (active) robust perception. These approaches typically involve probabilistic data association and hypothesis tracking given available data. Thus, for the object detection example, each hypothesis may represent a candidate object from a given database that the current observation (e.g. image or point-cloud) is successfully registered to. Similarly, one might reason probabilistically regarding perceptual aliasing, as in the first example above, which would also involve probabilistic data association. Yet, existing robust perception approaches focus on the passive case, where robot actions are externally determined and given, while the closely related approaches for active object detection and classification consider the robot to be perfectly localised.

In this work we develop a general data association aware belief space planning (DA-BSP) framework capable of better handling complexities arising in real world, possibly perceptually aliased, scenarios. We rigorously incorporate reasoning about data association within belief space planning, while also considering

other sources of uncertainty (motion, sensing and environment). In particular, we show our framework can be used for active disambiguation by determining appropriate actions, e.g. future viewpoints, for increasing confidence in a certain data association hypothesis.

Organization of the paper: After discussing related work and stating our contributions, we formulate the considered problem in Section 2. In Section 3 we provide concept overview and then discuss in detail the proposed approach, while demonstrating key aspects in simulated basic and realistic scenarios in Section 4. Finally, in Section 5 we conclude the discussion and suggest potential directions for future research.

1.1 Related Work

Calculating optimal solutions to POMDP is computationally intractable (PSPACE-complete) [22] for all but the smallest problems. The vast research area of approximate approaches (with reduced computational complexity) can be roughly segmented into point-based value iteration methods [19, 26], simulation based [30] and sampling based approaches [2, 6, 27], and direct trajectory optimization [11, 25, 33] methods. In all cases, finding the (locally) optimal actions involves evaluating a given objective function while considering future observations to be acquired as a result of each candidate action. They all assume DAS. For example, it is typically assumed that the robot can be localised by making observations of known landmarks or beacons (see, e.g. [2, 27]), while assuming to correctly associate each future measurement with an appropriate landmark. Though reasonable in certain scenarios, DAS becomes unrealistic in the presence of perceptually aliased environments (two scenes that look alike) and localisation uncertainty, as in this work.

The issue of perceptual aliasing has been considered in the earlier works on POMDP planning, though again with highly simplified scenarios, since the data-association further complicates the problem. In a slightly separate line of research, the approaches that study the issue were in the context of multiple hypothesis tracking (see [28] for earliest work on MHT) or more recently, of active robust perception. Both these approaches rely on passive and often non-parametric approaches, through various filtering techniques; we refer an interested reader to the book [15] and tutorial [3] for further details. For example, [34] proposed using Gaussian mixture probability hypothesis density (PHD) filter. To the best of our knowledge, such approaches are not considered in the context of active planning.

Coming back to scalable planning methods such as BSP, we note that while the traditional BSP approaches had typically assumed the environment to be accurately known (e.g. a given map), recent works, including [8, 9, 11, 18, 35], relax this assumption and model the uncertainty of the environment mapped thus far within the belief. The corresponding framework is thus tightly related to active SLAM, with the well known trade-off between exploration and exploitation. Recent work [9, 11, 18, 35] in this branch focused in particular on probabilistically

modelling what future observations will be obtained given a candidate action. Though none of them relax DAS assumption.

In the last few years, the SLAM research community has investigated approaches to be resilient to false data association (outliers) overlooked by front-end algorithms (e.g. image matching), see e.g. [7, 13, 14, 21, 31]. However these approaches, also known as robust graph optimization approaches, are developed only for the passive problem setting, i.e. robot actions are given and externally determined. In contrast, we consider a complimentary *active* framework that incorporates data association aspects within BSP.

Our approach is also tightly related with recent work on active hypothesis disambiguation in the context object detection and classification [4, 20, 29, 32, 36]. Given hypotheses regarding object class and pose, these approaches aim to find a sequence future viewpoints that will lead to disambiguation, i.e. identifying the correct hypothesis. However, these approaches assume the sensor is perfectly localized and can be shown to be a specific case of DA-BSP.

Probably the closest work to our approach is by Agarwal et al. [1], where the authors also consider hypotheses due to ambiguous data association and develop a BSP approach for active disambiguation. However, unlike them, DA-BSP considers ambiguous data association also in posterior and thus does not require a guarantee of fully disambiguating action in the future.

1.2 Contributions

To summarize, our main contributions in this paper¹ are as follows: (a) relaxing the data-association-is-solved assumption for a general data-association aware BSP framework (DA-BSP) with GMM priors (b) considering active data-association aspect for both planning and inference, hence providing a closed-loop framework (c) reducing some of the known recent BSP approaches to a degenerate cases of DA-BSP (d) demonstrating empirical results in support of two claims: data-association is crucial for a robust BSP and the principled approach of DA-BSP can be scalable enough to be applied on practical problems.

2 Notations and Problem Formulation

Consider a robot operating in a partially known or pre-mapped environment which can be ambiguous and perceptually aliased. The robot takes observations of different scenes and objects in the environment, and uses these observations to infer application-dependent random variables of interest (e.g. past and current robot poses). The following three spaces are involved in the considered problem, as shown in Figure 1: *pose-space*, *scene-space* and *observation-space*.

Pose-space involves all possible perspectives a robot can take with respect to a given environment model and in the context of task at hand. We denote the robot pose at time step k by x_k and a sequence of poses from 0 up to k by

¹Earlier versions of this paper appeared in [24] and [23].

$X_k \doteq \{x_0, \dots, x_k\}$. Given all controls $u_{0:k-1} \doteq \{u_0, \dots, u_{k-1}\}$ and observations $Z_{0:k} \doteq \{Z_0, \dots, Z_k\}$ up to time step k , the posterior probability distribution function² is defined as $\mathbb{P}(X_k|u_{0:k-1}, Z_{0:k})$. For notational convenience, we define below histories \mathcal{H}_k and \mathcal{H}_{k+1}^- and rewrite the posterior pdf (*belief*), at time k as $b[X_k] \doteq \mathbb{P}(X_k|\mathcal{H}_k)$.

$$\mathcal{H}_k \doteq \{u_{0:k-1}, Z_{0:k}\} \quad , \quad \mathcal{H}_{k+1}^- \doteq \mathcal{H}_k \cup \{u_k\}. \quad (1)$$

The *scene-space* involves a discrete set of objects or scenes, denoted by the set $\{A_{\mathbb{N}}\}$, in the given world model, and which can be detected through the sensors of the robot. We will use symbols A_i and A_j to denote such typical scenes. Note that even if the objects are identical, they are distinct in scene space. This will be important when we shall consider the cases where the objects look similar from some perspectives. Finally, *observation-space* is the set of all possible observations that the robot is capable of obtaining when considering its mission and sensory capabilities.

We consider probabilistic motion and observation models

$$x_{k+1} = f(x_k, u_k) + w_k \quad , \quad z_k = h(x_k, A_i) + v_k, \quad (2)$$

and denote them by $\mathbb{P}(x_{k+1}|x_k, u_k)$ and $\mathbb{P}(z_k|x_k, A_i)$, respectively. As common in literature, we consider Gaussian zero-mean process and measurement noise $w_i \sim \mathcal{N}(0, \Sigma_w)$ and $v_k \sim \mathcal{N}(0, \Sigma_v)$, with known noise covariance matrices Σ_w and Σ_v . Here, $h(x_k, A_i)$ is a noise-free observation which we would refer as *nominal* or *predicted* observation \hat{z} , that corresponds to observing scene A_i from pose x_k .

Given a prior $\mathbb{P}(x_0)$ and motion and observation models (2), the joint posterior pdf at the current time k can be written as

$$\mathbb{P}(X_k|\mathcal{H}_k) = \mathbb{P}(x_0) \prod_{i=1}^k \mathbb{P}(x_i|x_{i-1}, u_{i-1}) \mathbb{P}(Z_i|x_i, A_i). \quad (3)$$

Note that DAS is the underlying assumption in the above equation.

If the prior $\mathbb{P}(x_0)$ is Gaussian, it is not difficult to show that $b[X_k]$ is also a Gaussian with some mean \hat{X}_k and covariance Σ_k that can be efficiently calculated via maximum a posteriori (MAP) inference, see e.g. [17]. It is also valid in case where the environment model is given but uncertain, and when this model is unknown a priori and instead is constructed on-line within SLAM framework. However, in this paper we consider a more general case where the prior belief is modeled by a Gaussian mixture model (GMM). Such a situation can arise, for example, in the kidnapped robot problem in a perceptually aliased environment (e.g. different similar in appearance rooms), where matching sensor observations against a given map would indicate several most probable robot locations. In

²Strictly speaking, this is either the probability mass function or the probability density function for a discrete or a continuous random variable, respectively.

such a case the belief at time k can be represented by a GMM,

$$b[X_k] = \sum_{j=1}^{M_k} \xi_k^j \mathbb{P}(X_k | \mathcal{H}_k, \gamma = j), \quad (4)$$

where M_k is the number of components (or modes), the j th component is represented by the weight $\xi_k^j \doteq \mathbb{P}(\gamma = j | \mathcal{H}_k)$, modeling the probability of the robot being in that component, and by the conditional Gaussian

$$b[X_k^j] \doteq \mathbb{P}(X_k | \mathcal{H}_k, \gamma = j) = \mathcal{N}(\hat{X}_k^j, \Sigma_k^j), \quad (5)$$

with appropriate mean \hat{X}_k^j and covariance Σ_k^j . Here, γ is an indicator variable denoting the component number.

Given the belief at time k , one can reason about the robot's best future actions that would minimize (or maximize) an objective function J .

$$J(u_k) = \mathbb{E} \{c(b[X_{k+1}], u_k)\}, \quad (6)$$

where the expectation is over the (unknown) future observation z_{k+1} , and $c(\cdot)$ is the immediate cost.

The posterior belief at time t_{k+1} is a function of control u_k and observation z_{k+1} , i.e.

$$b[X_{k+1}] \doteq \mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}) \equiv \mathbb{P}(X_{k+1} | \mathcal{H}_k, z_{k+1}, u_k). \quad (7)$$

Note that, according to Eq. (6), we need to calculate the posterior belief (7) for *each* possible value of z_{k+1} .

Similarly, we define the propagated joint belief as

$$b[X_{k+1}^-] \doteq \mathbb{P}(X_k | \mathcal{H}_k) \mathbb{P}(x_{k+1} | x_k, u_k), \quad (8)$$

from which the marginal belief over the future pose x_{k+1} can be calculated as $b[x_{k+1}^-] \doteq \int_{-x_{k+1}} b[X_{k+1}^-]$.

In particular, the propagated belief at the first look ahead step, given the GMM belief (4) at time k is

$$b[X_{k+1}^-] = \sum_{j=1}^{M_k} \xi_{k+1}^{j-} b[X_{k+1}^{j-}], \quad (9)$$

with $\xi_{k+1}^{j-} \doteq \mathbb{P}(\gamma_{k+1} = j | \mathcal{H}_{k+1}^-) \equiv \xi_k^j$, and $b[X_{k+1}^{j-}] \doteq \mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, \gamma_k = j) = b[X_k^j] \mathbb{P}(x_{k+1} | x_k, u_k)$.

As earlier, with DAS assumption, one can consider for each specific value of z_{k+l} the corresponding observed scene A_i , and express the posterior (7) recursively as

$$b[X_{k+1}] = \eta b[X_{k+1}^-] \mathbb{P}(z_{k+1} | x_{k+1}, A_i), \quad (10)$$

which can be represented as $b[X_{k+1}] = \mathcal{N}(\hat{X}_{k+1}, \Sigma_{k+1})$ with appropriate mean \hat{X}_{k+1} and covariance Σ_{k+1} . The optimal control is then defined as:

$$u_k^* \doteq \arg \min_{u_k} J(u_k).$$

DAS assumption simplifies greatly the above formulation. Yet, in practice, determining data association reliably is often a non trivial task by itself, especially when operating in perceptually aliased environments. An incorrect data association (wrong scene A_i in Eq. (10)) can lead to catastrophic results, see, e.g. [12–14]. In this work we relax this restricting assumption and rigorously incorporate data association aspects within belief space planning and inference considering the underlying distributions are GMMs.

3 Approach

Given some candidate action (or sequence of actions) and the belief at planning time k , we can reason about a future observation z_{k+1} (e.g. an image) to be obtained once this action is executed; its actual value is unknown. All the possible values such an observation can assume should be taken into account while evaluating the objective function; hence, the expectation operator in Eq. (6). When written explicitly it transforms to

$$J(u_k) \doteq \int_{z_{k+1}} \overbrace{\mathbb{P}(z_{k+1} \mid \mathcal{H}_{k+1}^-)}^{(a)} c \left(\overbrace{\mathbb{P}(X_{k+1} \mid \mathcal{H}_{k+1}^-, z_{k+1})}^{(b)} \right) \quad (11)$$

The two terms (a) and (b) in the above equation have intuitive meaning: for each considered value of z_{k+1} , (a) represents how likely is it to get such an observation when both the history \mathcal{H} and control u_k are known, while (b) corresponds to the posterior belief *given* this specific z_{k+1} .

Considering DAS means we can correctly associate each possible measurement z_{k+1} with the corresponding scene A_i it captures, as in Eq. (10). Yet, it is unknown from what future robot pose x_{k+1} the actual observation z_{k+1} will be acquired, since the *actual* robot pose x_k at time k is unknown and the control is stochastic. Indeed, as a result of action u_k , the robot actual (true) pose x_{k+1} can be anywhere within the propagated belief $b[x_{k+1}^-]$. In inference, we have a similar situation with the key difference that the observation z has been acquired. We must first associate the captured measurement z with the scene or object A_i it describes, i.e. write the appropriate measurement likelihood term in the posterior (3).

In BSP framework, solved data association means that for each such observation $z \in \{z\}$ the corresponding observed scene $A_i \in \mathcal{A}$ is known. In contrast, we do not assume this, and instead reason about possible scenes or objects that the future observation z_{k+1} could be generated from, see Figures 1b and 1.

Parsimonious data association: Incorporating data-association is expensive. However, if the environment has only distinct scenes or objects, then for each specific value of z_{k+1} , there will be only one scene A_i that can generate such an observation according to the model (2). In case of perceptually aliased environments, there could be also several scenes (or objects) that are either completely identical,

or have a similar visual appearance when observed from appropriate viewpoints. They could equally well explain the considered observation z_{k+1} . Thus, there are several possible associations $\{A_i\}$ and due to localisation uncertainty determining which association is the correct one is not trivial. As we show in the sequel, in these cases the posterior $b[X_{k+1}]$ (term (b) in Eq. (11)) becomes a Gaussian mixture with appropriate weights that we rigorously compute. Additionally, the weight updates are capable of discriminating against unlikely data-associations, during the planning steps.

Perceptual aliasing: Intuitively speaking, perceptual aliasing occurs when an object different from the actual one, produces the same observation and thereby is an alias, in the sense of perception, to the true object. Consider two notions of perceptual aliasing: *exact* and *probabilistic*. Exact perceptual aliasing of scenes A_i and A_j is defined as $\exists x, x', h(x, A_i) = h(x', A_j)$, and will be denoted in this paper by $\{A_i, A_j\}_{\text{aliased}}$. In other words, the same nominal (noise-free) observation \hat{z} can be generated by observing different scenes, possibly from different viewpoints. Such a situation is depicted in Figure 1. A probabilistic perceptual aliasing is a more general form of aliasing, which can be defined as $\exists x, x', |\mathbb{P}(z|A_i, x) - \mathbb{P}(z|A_j, x')| < \epsilon$ for some small threshold ϵ .

3.1 Computing the term (a) : $\mathbb{P}(z_{k+1}|\mathcal{H}_{k+1}^-)$

Applying total probability over non-overlapping scene space $\{A_{\mathbb{N}}\}$ and marginalizing over all possible robot poses, yields

$$\mathbb{P}(z_{k+1}|\mathcal{H}_{k+1}^-) \equiv \sum_i^{|A_{\mathbb{N}}|} \int_x \mathbb{P}(z_{k+1}, x, A_i | \mathcal{H}_{k+1}^-) \doteq \sum_i^{|A_{\mathbb{N}}|} w_{k+1}^i. \quad (12)$$

As seen from the above equation, to calculate the likelihood of obtaining some observation z_{k+1} , we consider separately, for each scene $A_i \in \{A_{\mathbb{N}}\}$, the likelihood that this observation was generated by scene A_i . This probability is captured for each scene A_i by a corresponding weight w_{k+1}^i ; these weights are then summed to get the actual likelihood of observation z_{k+1} . As will be seen below, these weights naturally account for perceptual aliasing aspects for each considered z_{k+1} .

In practice, instead of considering the entire scene space $\{A_{\mathbb{N}}\}$ that could be huge, the availability of the belief $b[X_{k+1}^-]$ makes it possible to consider only those scenes that could be actually observed from viewpoints with non-negligible probability according $b[X_{k+1}^-]$, e.g. within 3 standard deviations of uncertainty for each GMM component. In the following, however, we proceed while reasoning about the entire scene space $\{A_{\mathbb{N}}\}$.

Proceeding with the derivation further, using the chain rule we compute

$$\sum_i \int_x \mathbb{P}(z_{k+1} | x, A_i, \mathcal{H}_{k+1}^-) \mathbb{P}(A_i, x | \mathcal{H}_{k+1}^-) \quad (13)$$

However, since $\mathbb{P}(A_i, x \mid \mathcal{H}_{k+1}^-) = \mathbb{P}(A_i \mid x \mid \mathcal{H}_{k+1}^-)b[x_{k+1}^- = x]$, we get

$$\sum_i^{|A_{\mathbb{N}}|} \int_x \mathbb{P}(z_{k+1} \mid x, A_i, \mathcal{H}_{k+1}^-) \mathbb{P}(A_i \mid \mathcal{H}_{k+1}^-, x) b[x_{k+1}^- = x]. \quad (14)$$

Thus,

$$w_{k+1}^i \doteq \int_x \mathbb{P}(z_{k+1} \mid x, A_i, \mathcal{H}_{k+1}^-) \mathbb{P}(A_i \mid \mathcal{H}_{k+1}^-, x) b[x_{k+1}^- = x]. \quad (15)$$

Since the propagated belief (9), from which $b[x_{k+1}^-]$ is calculated, is a GMM, we can replace $b[x_{k+1}^- = x]$ with $\sum_{j=1}^{M_k} \xi_{k+1,j}^- b[x_{k+1,j}^- = x]$.

Here, $\mathbb{P}(z_{k+1} \mid A_i, x, \mathcal{H}_{k+1}^-) \equiv \mathbb{P}(z_{k+1} \mid A_i, x)$ is the standard measurement likelihood term, while $\mathbb{P}(A_i \mid \mathcal{H}_{k+1}^-, x)$ represents the *event likelihood*, which denotes the probability of scene A_i to be observed from viewpoint x . In other words, this scenario-dependent term encodes from what viewpoints each scene A_i is observable and could also model occlusion and additional aspects. As such, this term can be determined given a model of the environment and thus, in this work, we consider this term to be given.

The weights w_{k+1}^i (15) naturally capture *perceptual aliasing* aspects: consider some observation z_{k+1} and the corresponding generative model $z_{k+1} = h(x^{tr}, A^{tr}) + v$ with appropriate unknown *true* robot pose x^{tr} and scene $A^{tr} \in \{A_{\mathbb{N}}\}$. Clearly, the measurement likelihood $\mathbb{P}(z_{k+1} \mid x, A_i, \mathcal{H}_{k+1}^-)$ will be high when evaluated for $A_i = A^{tr}$ and in vicinity of x^{tr} . Note that we will necessarily consider such a case, since according to Eq. (12) we separately consider each scene A_i in $\{A_{\mathbb{N}}\}$, and, given A_i , we reason about all poses x in Eq. (15). In case of perceptual aliasing, however, there will be also another scene(s) A_j which could generate the same observation z_{k+1} from appropriate robot pose x' . Thus, the corresponding measurement likelihood term to A_j will also be high for x' .

However, the actual value of w_i (for each $A_i \in \{A_{\mathbb{N}}\}$) depends, in addition to the measurement likelihood, also on the mentioned-above event likelihood and on the GMM belief $b[x_{k+1}^-]$, with the latter weighting the probability of each considered robot pose x . This correctly captures the intuition that those observations z with low-probability poses $b[x_{k+1}^- = x^{tr}]$ will be unlikely to be actually acquired, leading to low value of w_i with $A_i = A^{tr}$. Since $b[x_{k+1}^-]$ is a GMM with M_k components, low-probability pose x^{tr} corresponds to low probabilities $b[x_{k+1}^j = x^{tr}]$ for each component $j \in \{1, \dots, M_k\}$. However, the likelihood term (12) could still go up in case of perceptual aliasing, where the aliased scene A_j generates a similar observation to z_{k+1} from viewpoint x' with latter being more probable, i.e. high probability $b[x_{k+1}^- = x']$.

In practice, calculating the integral in Eq. (15) can be done efficiently considering separately each component of the GMM $b[x_{k+1}^-]$. Each such component is a Gaussian that is multiplied by the measurement likelihood $\mathbb{P}(z_{k+1} \mid A_i, x, \mathcal{H})$ which is also a Gaussian and it is known that a product of Gaussians remains a Gaussian. The integral can then be only calculated for the window where event

likelihood is non-zero i.e $\mathbb{P}(A_i | x, \mathcal{H}) > 0$. For general probability distributions, the integral in Eq. (15) should be computed numerically. Since in practical applications $\mathbb{P}(A_i | x, \mathcal{H})$ is sparse w.r.t. x , this computational cost is not severe.

3.2 Computing the term (b) : $\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1})$

The term (b), $\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1})$, represents the posterior probability conditioned on observation z_{k+1} . This term can be similarly calculated, with a key difference: since the observation z_{k+1} is given, it must have been generated by *one* specific (but unknown) scene A_i according to measurement model (2). Hence, also here, we consider all possible such scenes and weight them accordingly, with weights \tilde{w}_{k+1}^i representing the probability of each scene A_i to have generated the observation z_{k+1} . As will be seen next, the posterior $\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1})$ is a GMM with M_{k+1} components.

Applying total probability over non-overlapping $\{A_{\mathbb{N}}\}$ and chain-rule, we get:

$$\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}) = \sum_{i=1}^{|A_{\mathbb{N}}|} \mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}, A_i) \cdot \mathbb{P}(A_i | \mathcal{H}_{k+1}^-, z_{k+1}). \quad (16)$$

The first term, $\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}, A_i)$, is the posterior belief conditioned on observation z_{k+1} , history \mathcal{H}_{k+1}^- , as well as a candidate scene A_i that supposedly generated the observation z_{k+1} . It is not difficult to show that this posterior is actually the GMM

$$\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}, A_i) = \sum_{j=1}^{M_k} \xi_k^j b[X_{k+1}^{j+} | A_i], \quad (17)$$

where $b[X_{k+1}^{j+} | A_i] \doteq \mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, \gamma = j, A_i, z_{k+1})$ is the posterior of the j th GMM component of the propagated belief $b[X_{k+1}^-]$, see Eq. (9).

Plugging in Eq. (17) back into $\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1})$ yields from Eq. (10):

$$b[X_{k+1}] \equiv \mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}) = \sum_{i=1}^{|A_{\mathbb{N}}|} \sum_{j=1}^{M_k} \xi_k^j \mathbb{P}(A_i | \mathcal{H}_{k+1}^-, z_{k+1}) b[X_{k+1}^{j+} | A_i]. \quad (18)$$

The term, $\mathbb{P}(A_i | \mathcal{H}_k, u_k, z_{k+1})$, is merely the likelihood of A_i being actually the one which generated the observation z_{k+1} . This term can be evaluated, in a similar fashion to Section 3.1, accounting for $b[x_{k+1}^{j-}]$ for each considered j th component as $\mathbb{P}(A_i | \mathcal{H}_{k+1}^-, z_{k+1}) = \int_x \mathbb{P}(A_i, x | \mathcal{H}_{k+1}^-, z_{k+1})$, and applying Bayes' rule yields

$$\tilde{w}_{k+1}^{ij} \doteq \eta' \int_x \mathbb{P}(z_{k+1} | A_i, x, \mathcal{H}_{k+1}^-) \mathbb{P}(A_i | \mathcal{H}_{k+1}^-, x) b[x_{k+1}^{j-} = x], \quad (19)$$

with $\eta' = 1/\mathbb{P}(z_{k+1} | \mathcal{H}_{k+1}^-)$. Note that for each component j , $\sum_i \tilde{w}_{k+1}^{ij} = 1$. Finally, we can re-write Eq. (18) as

$$\mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}) = \sum_{r=1}^{M_{k+1}} \xi_{k+1}^r \mathbb{P}(X_{k+1} | \mathcal{H}_{k+1}, \gamma = r), \quad (20)$$

or in short, $b[X_{k+1}] = \sum_{r=1}^{M_{k+1}} \xi_{k+1}^r b[X_{k+1}^{r+}]$, where

$$\xi_{k+1}^r \doteq \xi_{k+1}^{ij} \equiv \xi_k^j \tilde{w}_{k+1}^{ij} \quad , \quad b[X_{k+1}^{r+}] \doteq b[X_{k+1}^{j+} | A_i]. \quad (21)$$

As seen, we got a new GMM with M_{k+1} components, where each component $r \in [1, M_{k+1}]$, with appropriate mapping to indices (i, j) from Eq. (18), is represented by weight ξ_{k+1}^r and posterior conditional belief $b[X_{k+1}^{r+}]$. The latter can be evaluated as the Gaussian $b[X_{k+1}^{r+}] \propto b[X_{k+1}^{j-}] \mathbb{P}(z_{k+1} | x_{k+1}, A_i) = \mathcal{N}(\hat{X}_{k+1}^r, \Sigma_{k+1}^r)$, where the mean \hat{X}_{k+1}^r and covariance Σ_{k+1}^r can be efficiently recovered via MAP inference.

3.3 Summary thus Far

To summarize the discussion thus far, we have shown that for the myopic case, the objective function (11) can be re-written as

$$J(u_k) = \int_{z_{k+1}} \left(\sum_i^{|A_{\mathbb{N}}|} w_{k+1}^i \right) \cdot c \left(\sum_r^{M_{k+1}} \xi_{k+1}^r b[X_{k+1}^{r+}] \right). \quad (22)$$

One can observe that according to Eq. (18), each of the M_k components from the belief at a previous time, is split into $|A_{\mathbb{N}}|$ new components with appropriate weights. This would imply an explosion in the number of components, making the proposed framework hardly applicable. However, in practice, the majority of the weights will be negligible, and therefore can be pruned, while the remaining number of components is denoted by M_{k+1} in Eq. (20). Depending on the scenario and the degree of perceptual aliasing, this can correspond to *full* or *partial* disambiguation.

Having shown incorporating data association within belief space planning leads to Eq. (22), we now proceed with the exposition of our approach.

3.4 Simulating Future Observations $\{z_{k+1}\}$ given $b[X_{k+1}^-]$

Calculating the objective function (22) for each candidate action u_k involves considering all possible realizations of z_{k+1} . One approach to perform this in practice, is to simulate future observations $\{z_{k+1}\}$ given propagated GMM belief $b[X_{k+1}^-]$, scenes $\{A_{\mathbb{N}}\}$ and observation model (2). One can then evaluate Eq. (22) considering all observations in $\{z_{k+1}\}$.

We now briefly describe how this concept can be realised. First, viewpoints $\{x\}$ are sampled from $b[X_{k+1}^-]$. For each viewpoint $x \in \{x\}$, an observed scene

A_i is determined according to event likelihood $\mathbb{P}(A_i \mid \mathcal{H}_k, x)$. Together, x and A_i are then used to generate nominal $\hat{z} = h(x, A_i)$ and noise-corrupted observations $\{z\}$ according to observation model (2): $z = h(x, A_i) + v$. The set $\{z_{k+1}\}$ is then the union of all such generated observations $\{z\}$. Note that while generating $\{z_{k+1}\}$, the true association is known (scene A_i), it is unknown to our algorithm, i.e. while evaluating Eq. (22).

3.5 Computing Mixture of Posterior Beliefs $\sum_i \tilde{w}_i b[X_{k+1}^{i+}]$

As seen from Eq. (22), reasoning about data association aspects resulted in a mixture of posteriors within the cost $c(\cdot)$, i.e. $\sum_i \tilde{w}_i b[X_{k+1}^{i+}]$, for each possible observation $z_{k+1} \in \{z_{k+1}\}$. In this section we briefly describe how one can actually calculate the corresponding posterior distributions, given some specific observation $z_{k+1} \in \{z_{k+1}\}$. For simplicity, we consider the belief at planning time k is a Gaussian $b[X_k] = \mathcal{N}(\hat{X}_k, \Sigma_k)$. However, our approach could be applied also to more general cases (e.g. mixture of Gaussians) with a certain price in terms of computational complexity. Further investigation of these aspects is left to future research.

Under this setting, each of the components $b[X_{k+1}^{i+}]$ in the mixture pdf can be written as $b[X_{k+1}^{i+}] \propto b[X_k] \mathbb{P}(x_{k+1} \mid x_k, u_k) \mathbb{P}(z_{k+1} \mid x_{k+1}, A_i)$. It is then not difficult to show that the above belief is a Gaussian $b[X_{k+1}^{i+}] = \mathcal{N}(\hat{X}_{k+1}^i, \Sigma_{k+1}^i)$ and to find its first two moments via MAP inference. Obviously, the mixture of posterior beliefs in the cost $c(\cdot)$ from Eq. (22) is now a mixture of Gaussians:

$$\sum_i \tilde{w}_i b[X_{k+1}^{i+}] = \sum_i \tilde{w}_i \mathcal{N}(\hat{X}_{k+1}^i, \Sigma_{k+1}^i). \quad (23)$$

3.6 Designing a Specific Cost Function

The treatment so far has been agnostic to the structure of the cost function $c(\cdot)$. Recalling Eq. (22) we see that the belief over which the cost function is defined, is multimodal in general. Standard cost functions in literature, typically include terms such as control usage c_u , distance to goal c_G and uncertainty c_Σ , see e.g. [11, 33]. In our case, however, the specific form of the latter should be re-examined and an additional term quantifying ambiguity level can be introduced. In this section we thus briefly discuss these two terms, starting with the cost over posterior uncertainty.

Since, unlike in usual BSP, the posterior belief in our case is multimodal and represented as mixture of Gaussians $\sum_i \tilde{w}_i \mathcal{N}(\hat{X}_{k+1}^i, \Sigma_{k+1}^i)$, see Eq. (23), we could define several different cost structures depending on how we treat the different modes. Two particular such costs are taking the worst-case covariance among all covariances Σ_{k+1}^i in the mixture, e.g. $\Sigma = \max_i \{tr(\Sigma_i)\}$, or to collapse the mixture into a single Gaussian $\mathcal{N}(\cdot, \Sigma)$, see e.g. [5]. In both cases, we can define the cost due to uncertainty as $c_\Sigma = trace(\hat{\Sigma})$.

The cost due to ambiguity, c_w , should penalise ambiguities such as those arising out of perceptual aliasing. Here, we note that non-negligible weights w_i in Eq. (22) arise due to perceptual aliasing, whereas in case of no aliasing, all but one of these weights are zero. In most severe case of aliasing (all scenes or objects A_i are identical), all of these weights are comparable among each other. Thus we take Kullback-Leibler divergence $KL_u(\{\tilde{w}_i\})$ of these weights $\{\tilde{w}_i\}$ from a uniform distribution to penalise higher aliasing, and define $c_w(\{\tilde{w}_i\}) \doteq \frac{1}{KL_u(\{\tilde{w}_i\}) + \epsilon}$, where ϵ is a small number to avoid division-by-zero in case of extreme perceptual aliasing. With user-defined weights M_u, M_G, M_Σ and M_w , the overall cost then can be defined as a combination

$$c \doteq M_u c_u + M_G c_G + M_\Sigma c_\Sigma + M_w c_w, \quad (24)$$

3.7 Formal Algorithm for DA-BSP

We now have all the ingredients to present the overall framework of data-association aware belief space planning, calling it DA-BSP for brevity. It is summarised in Algorithm 1 and briefly described below.

Given a GMM belief $b[X_k]$ and candidate action u_k , we first propagate the belief to get $b[X_{k+1}^-]$ and then simulate future observations $\{z_{k+1}\}$ (line 2). The algorithm then calculates the contribution of each observation $z_{k+1} \in \{z_{k+1}\}$ to the objective function (22). In particular, on lines 8 and 9 we calculate the weights w_{k+1}^i that are used in evaluating the likelihood w_s of obtaining observation z_{k+1} . On lines 10-16 we compute the posterior belief: this involves updating each j th component from the propagated belief $b[X_{k+1}^{j-}]$ with observation z_{k+1} , considering each of the possible scenes A_i . After pruning (line 18), this yields a posterior GMM with M_{k+1} components. We then evaluate the cost $c(\cdot)$ (line 20) and use w_s to update the value of the objective function J with the weighted cost for measurement z_{k+1} (line 21).

4 Experimental results

4.1 An Abstract Example for DA-BSP

Consider the problem of robotic manipulation of objects in the kitchen. For simplicity, let us abstract it to a simpler domain of three objects, $|\{A_N\}| = 3$. We consider a single step control at time step k , from a given belief $b[X_k]$, as well as that of one step ahead $b[X_{k+1}^-]$, and assume the following motion and observation models f and h

$$\begin{aligned} f(x, u) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot x + d \begin{cases} [0, 1]^T & \text{if } u = \textit{up} \\ [1, 0]^T & \text{if } u = \textit{right} \end{cases}, \\ h(x, A_i) &= h_i(x) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot (x - x_i) + s_i. \end{aligned} \quad (25)$$

Algorithm 1 Data association aware belief-space planning

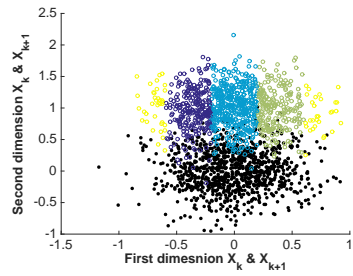
Input: Current GMM belief $b[X_k]$ at step- k , history \mathcal{H}_k , action u_k , scenes $\{A_N\}$, event likelihood $\mathbb{P}(A_i \mid \mathcal{H}_k, x)$ for each $A_i \in \{A_N\}$

```
1:  $b[X_{k+1}^-] \leftarrow b[X_k]\mathbb{P}(x_{k+1} \mid x_k, u_k)$  ▷ Eq. (9)
2:  $\{z_{k+1}\} \leftarrow \text{SimulateObservations}(b[X_{k+1}^-], \{A_N\})$ 
3:  $J \leftarrow 0$ 
4: for  $\forall z_{k+1} \in \{z_{k+1}\}$  do
5:    $w_s \leftarrow 0$ 
6:   for  $i \in [1 \dots |A|]$  do
7:     ▷ compute weight, Eq. (15)
8:      $w_{k+1}^i \leftarrow \text{CalcWeights}(z_{k+1}, \mathbb{P}(A_i \mid \mathcal{H}_{k+1}^-, x), b[X_{k+1}^-])$ 
9:      $w_s \leftarrow w_s + w_i$ 
10:    for  $\forall j \in [1, \dots, M_k]$  do
11:      ▷ compute weight  $\tilde{w}_{k+1}^{ij}$  for each GMM component, Eq. (19)
12:       $\tilde{w}_{k+1}^{ij} \leftarrow \text{CalcWeights}(z_{k+1}, \mathbb{P}(A_i \mid \mathcal{H}_{k+1}^-, x), b[X_{k+1}^{j-}])$ 
13:       $\xi_{k+1}^{ij} \leftarrow \xi_k^j \tilde{w}_{k+1}^{ij}$  ▷ Eq. (21)
14:      ▷ Calculate posterior of  $b[X_{k+1}^{j-}]$ , given  $A_i$ 
15:       $b[X_{k+1}^{ij+}] \leftarrow \text{UpdateBelief}(b[X_{k+1}^{j-}], z_{k+1}, A_i)$ 
16:    end for
17:  end for
18:  Prune components with weights  $\xi_{k+1}^{ij}$  below a threshold
19:  Construct  $b[X_{k+1}^+]$  from the remaining  $M_{k+1}$  components via Eq. (20)
20:   $c \leftarrow \text{CalcCost}(b[X_{k+1}^+])$  ▷ Eq. 24
21:   $J \leftarrow J + w_s \cdot c$ 
22: end for
23: return  $J$ 
```

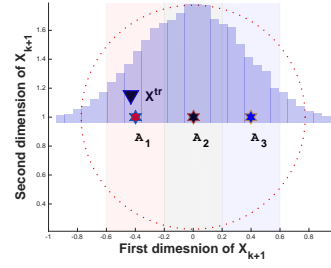
where observations as well as the shift s_i is in an object-centric frame, with x_i representing location of A_i . Intuitively, s_i is a simple mechanism to model perceptual aliasing between objects; e.g., identical objects A_i would have the same s_i . Figure 2 illustrates the process of simulating future observations $\{z_{k+1}\}$ for $u_k = up$, considering unique and perceptually aliased scenes (Figures 2c-2d). In particular, a sampled pose x^{tr} used to generate an observation $z_{k+1} \in \{z_{k+1}\}$ is shown in Figure 2b.

Figure 3 demonstrates key aspects in our approach, considering each time a single observation z_{k+1} . Our approach reasons about data association and hence we consider each z_{k+1} could have been generated by one of the 3 objects; each such association would fetch us a conditional posterior belief $b[X_{k+1}^{i+}]$ as denoted by small ellipses. Finally, we compute the total cost according to Algorithm 1.

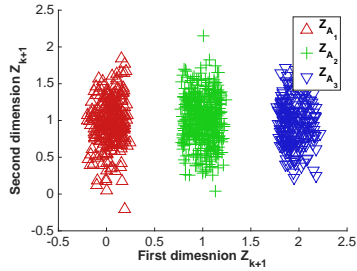
Figures 3a-3d denote the situation when the true pose x^{tr} is close to center and observe A_2 , while in Figures 3e-3h it is at the left side and observe A_1 . Different degrees of aliasing are considered. Both weights w_i and \tilde{w}_i are shown in the inset histograms. Note that the unnormalised weight w_i is higher when the object is at the centre, because the overall likelihood of the observation is higher. Also, with no aliasing, for any other scene A_j than the true one, the normalised weight w_j is small irrespective of where x^{tr} is. In other words, weights are also related to how likely the objects are to be the causes behind an observation; in case of no aliasing, this can be negligibly small. This is crucial since it implies



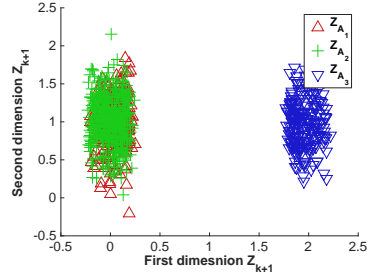
(a) Sampled viewpoints



(b) Event likelihood $\mathbb{P}(A_i|x, \mathcal{H}) \forall i$



(c) No aliasing, $\{\Phi\}_{\text{aliased}}$



(d) $\{A_1, A_3\}_{\text{aliased}}$

Fig. 2: Pose and observation space. (a) black-colored samples $\{x_k\}$ are drawn from $b[X_k] \doteq \mathcal{N}([0, 0]^T, \Sigma_k)$, from which, given control u_k , samples $\{x_{k+1}\}$ are computed, colored according to different scenes A_i being observed, and used to generate observations $\{z_{k+1}\}$. (b) Stripes represent locations from which each scene A_i is observable, histogram represents distribution of $\{x_{k+1}\}$, which corresponds to $b[X_{k+1}^-]$. (c)-(d) distributions of $\{z_{k+1}\}$ without aliasing and when $\{A_1, A_3\}_{\text{aliased}}$.

that DA-BSP in practical applications with infrequent aliasing, would not require any significant additional computational effort w.r.t. usual BSP.

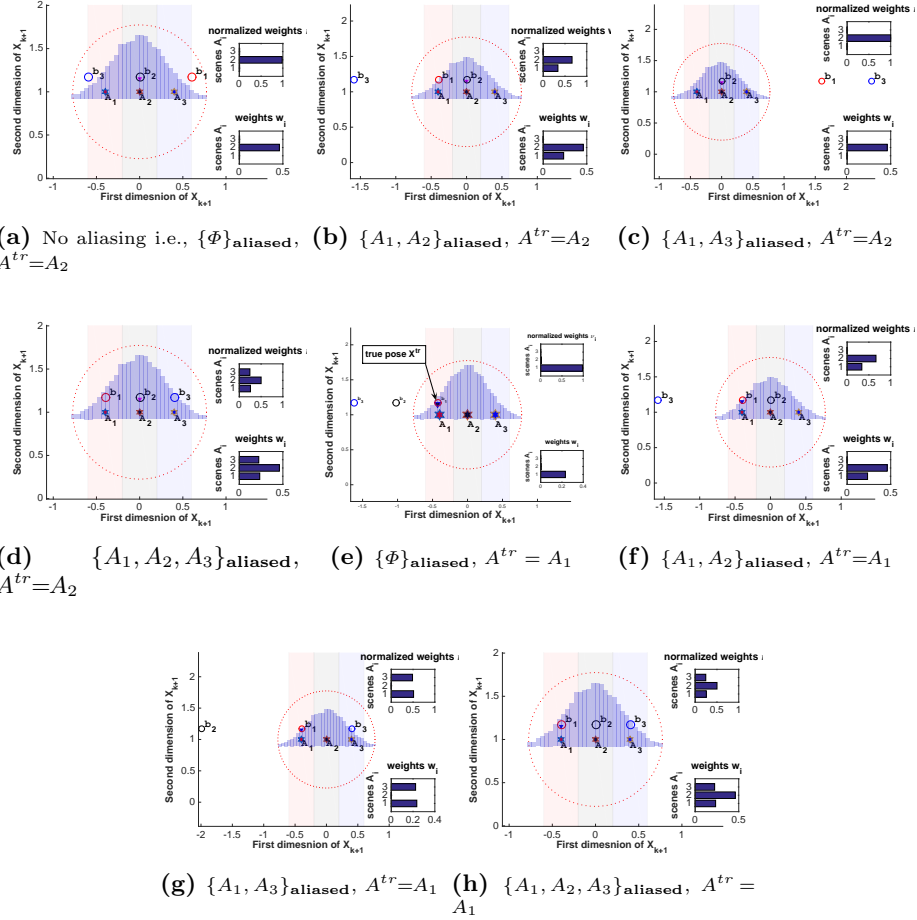


Fig. 3: DA-BSP for a single observation z_{k+1} . Red-dotted ellipse denotes $b[X_{k+1}^-]$, while the true pose that generated z_{k+1} is shown by inverted triangle. Smaller ellipses are the posterior beliefs $b[X_{k+1}^+]$. (a-d) x^{tr} is near center, observing A_2 ; (e-f) x^{tr} is on the left, observing A_1 . Weights w_i and \tilde{w}_i , corresponding to each scene A_i are shown in the inset bar-graphs.

Figures 3b-3d depict $\{A_1, A_2\}_{\text{aliased}}$, $\{A_1, A_3\}_{\text{aliased}}$ and $\{A_1, A_2, A_3\}_{\text{aliased}}$. When $\{A_1, A_3\}_{\text{aliased}}$, the weights w_i are similar, and indeed our cost c_w of weights (in Eq. (24)) is high. For similar uncertainty in pose, this cost would remain constant. Hence, in the presence of identical objects placed similarly within the current belief, optimization of general cost function would be guided towards *active localization*. On the other hand, if one object j lies closer to the

current nominal pose, it will have slightly higher w_j . In case $\{A_1, A_2, A_3\}_{\text{aliased}}$, i.e. *all* objects are identical, the weights w_i are simply an indication of the prior. This is reasonable since in such a case, considering different data association does not yield any new information.

Finally, in Table 1, we present the numerical analysis of cost computation (see Eq. (24)) of these configurations, as well as a metric $\{\epsilon_{BSP}, \epsilon_{DA}\}$ quantifying estimation error, defined over incorrect (w.r.t ground-truth) associations through random sampling of various modes. Intuitively ϵ_{BSP} and ϵ_{DA} evaluate how *good* the posterior mean is w.r.t. ground-truth x^{tr} for usual BSP and DA-BSP respectively (lower is better). Recall that unlike action u_1 , action u_2 leads to fully unambiguous observations, around most-likely value (see Fig. 3) and consequently, $\epsilon_{BSP} \simeq \epsilon_{DA}$.

config	cost				est. err.		\mathcal{U} u_1/u_2
	worst	max-wt.	KL_u	mode	ϵ_{BSP}	ϵ_{DA}	
$\{\Phi\}_{\text{aliased}}$	0.0977	0.0977	0.3496	1.1000	1.0851	0.1717	u_1
$\{\Phi\}_{\text{aliased}}$	0.1009	0.1009	-na-	1.0000	0.3461	0.3999	u_2
$\{A_1, A_2, A_3\}_{\text{aliased}}$	0.0508	0.0508	0.5072	3.0000	1.1654	0.4772	u_1
$\{A_1, A_2, A_3\}_{\text{aliased}}$	0.1009	0.1009	-na-	1.0000	0.3832	0.3990	u_2
$\{A_1, A_2\}_{\text{aliased}}$	0.0833	0.0833	0.3757	1.5500	1.2197	0.2114	u_1
$\{A_1, A_2\}_{\text{aliased}}$	0.1009	0.1009	-na-	1.0000	0.3912	0.3992	u_2
$\{A_1, A_3\}_{\text{aliased}}$	0.0849	0.0849	0.3649	1.4000	1.0552	0.4197	u_1
$\{A_1, A_3\}_{\text{aliased}}$	0.1009	0.1009	-na-	1.0000	0.4101	0.3940	u_2

Table 1: Evaluating different cost functions for various configurations (see Fig. 3)

4.2 Gazebo World

To demonstrate generality of DA-BSP, we compared (in simulation) it with current state of the art (denoted as BSP) and the approach proposed in [1]. For the latter case (see Fig. 4b), we have a simulated environment of rectangular corridors with shelves(s_i) and elevator(e), where a pioneer robot has a non-Gaussian belief prior (shown with p_1, p_2); as it can be in either of the two corridors, with localization as its objective. We evaluate our algorithm for inference (`infer`) as well as active planning (`plan`) (see Table 2). The absence of pose uncertainty, which is the case in [1], would lead us to a wrong inference and estimate the robot to be at pose p_1 (corridor 1) initially, whereas our approach which considers pose uncertainty will lead to correct inference with high weight for being in corridor 2 (Fig. 4b).

To compare with the current state of the art, we consider another scenario with 4 floors. Floor 1 has the same configuration as in Fig. 4b, floor 2 and floor 3 have the left and the right shelves (w.r.t floor 1) removed respectively and floor 4 has no shelves at all. We use the metric η_{da} as one of the possible

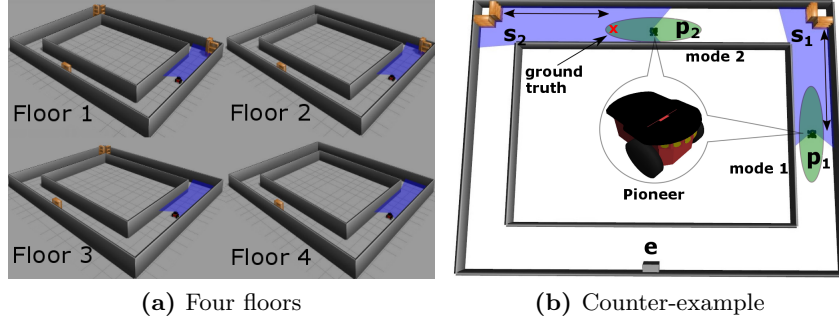


Fig. 4: Using Pioneer robot in Gazebo simulation. (a) four-floor aliased world. (b) counter-example for hypothesis reduction in absence of pose-uncertainty in prior. The inference incorrectly deduces that the robot is in mode 1.

ways to quantify data association performance by computing the probability of picking the right mode in the posterior GMM (which corresponds to ground-truth position of robot). Thus, for two equally weighted GMM components, $\eta_{da} = 0.5$ or 0 if the correct component is one of them or if it has been pruned. The number of modes in the posterior is also indicated (for planning, the total number of modes for *all* the considered future observations is shown). As seen, many of the data associations considered within BSP are wrong ($\eta_{da} = 0.29$), while also in inference an incorrect association is made ($\eta_{da} = 0$). This can lead to (possibly catastrophic) mission failure; in this case, failure to associate to the correct corridor. We also show a counter-example (Fig. 4b)) for [1] ($\eta_{da} = 0$) since that approach does not model uncertainty within each of the GMM components. In contrast, DA-BSP outperforms both approaches within planning and inference.

	config	cost		metrics	
		KL_u	Worst-Cov	modes	η_{da}
compare	DA-BSP plan	2.60	5.48	21	0.41
	DA-BSP infer	8.14	5.08	4	0.26
	BSP plan	-8.67	5.36	13	0.29
	BSP infer	-4.35	2.95	2	0
	[1] plan	-na-	-na-	-na-	-na-
	[1] infer	-63.76	2.82	2	0

Table 2: Evaluating DA-BSP

In order to evaluate DA-BSP, we consider a simplistic set of actions, namely $\{fwd_1, fwd_2, bwd_1\}$ for a one-step forward, two-step forward and one-step backward movements, respectively (see Table 3). These actions highlight the challenges of data-association aware planning, even in the context of a simplistic scenario. Note that number of modes (which signifies different associations planner is considering at that step) is significantly higher in the planning, than in the inference. However, when there exists a disambiguating action, such as fwd_2 is,

the planner is able to associate to the correct association all the time (demonstrated by $\eta_{da} = 1$).

config		cost		metrics	
		KL_u	Worst-Cov	modes	η_{da}
DA-BSP	bwd_1 plan	6571.29	28.74	48	0.08
	bwd_1 infer	6567.86	30.53	4	0.08
	$ fwd_1$ plan	-1160.93	6.22	22	0.18
	$ fwd_1$ infer	-1300.72	6.98	2	0.16
	$ fwd_2$ plan	-166.03	0.66	2	1
	$ fwd_2$ infer	-227.03	0.91	1	1

Table 3: DA-BSP for candidate actions

4.3 Aliased Multi-Floor Environment

Since, incorporating data-association implies that (at least theoretically) an exponential blow-up of number of unimodal beliefs maintained in the posterior, we therefore evaluated DA-BSP in simple but real-world scene by deploying a real Pioneer robot in a 3-floor aliased environment (see Fig. 5a), with similar objective of floor and position disambiguation. When not reasoning about perceptual aliasing, the robot takes greedy (w.r.t. control cost and position uncertainty) action and fails to disambiguate, whereas DA-BSP successfully tackles planning with data-association, as shown in Figure 5b. Although not witnessed here, BSP can not guarantee global optimum solution (w.r.t. control and uncertainty cost) and DA-BSP similarly can not provide a global least cost path for full disambiguation - a problem known to be NP-hard [10].

5 Conclusions

State-of-the-art belief space planning (BSP) approaches typically consider data association to be given and perfect. However, such an assumption is less appropriate in presence of localisation uncertainty while operating in ambiguous environments, where two scenes could be similar in appearance when observed from appropriate viewpoints. In this work, we developed a data association aware belief space planning (DA-BSP) approach that relaxes the aforementioned assumption. Our framework rigorously incorporates data association aspects within BSP, while considering different sources of uncertainty (uncertainty in robot motion, sensing and possibly in the observed environment). As such, it is capable of better coping with ambiguous, perceptually aliased, situations by appropriately calculating belief evolution and expected cost due to candidate actions, and in particular, could be used for active disambiguation. Thanks to this association being inherent in planning, DA-BSP considers data-association parsimoniously and a simple thresholding is enough for a scalable application of data-association aware belief space planning. We demonstrated key aspects of

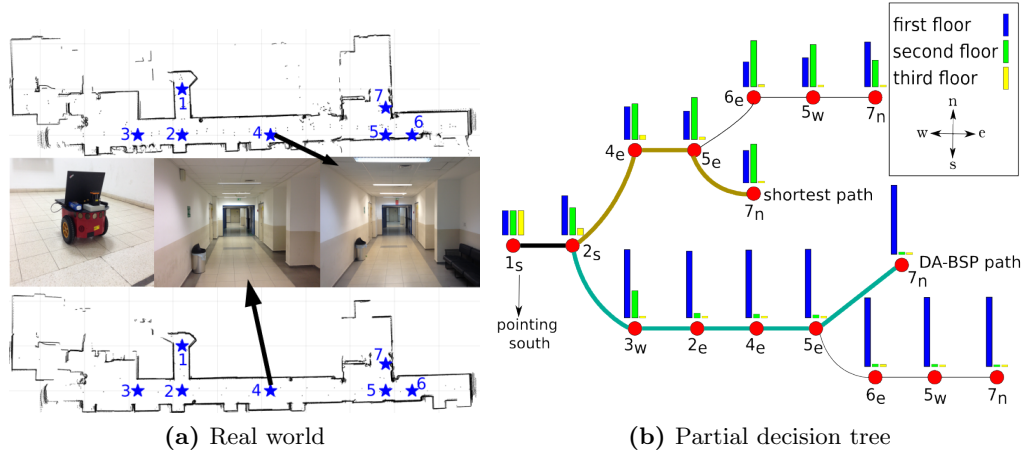


Fig. 5: Using Pioneer robot in the real-world. (a) two (of three) severely-aliased floors, and belief space planning for it (b) DA-BSP can plan for fully disambiguating path (otherwise sub-optimal, due to path-length) while usual BSP with *maximum likelihood* assumption can not (as shown by almost equi-probable modes in the histogram).

DA-BSP in abstract example as well as Gazebo simulations. We also applied it on a real-world problem with Pioneer robot lost in a multi-storied building.

One of the major contributions of this work is in proposing a data-association-aware robust perception in a unified framework of plan-infer-execute. This is in contrast with passive approaches known in robust perception literature as well as that of multi-hypothesis tracking. Consequently, we are currently looking into extending the approach to non-myopic setting such that the generality of the framework becomes further explicit. Additionally, proving the general theoretical properties of DA-BSP, such as *probabilistic completeness under uncertainty* along the lines proposed by [2], is another interesting direction of research. Apart from this, evaluating the approach in a more complex real-world scenarios is also an avenue for future research.

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